# IB Math 12 HL: Analysis and Approaches Course Outline 

Teachers: Mr. Sherry Mr. Wong<br>Email:<br>msherry@sd43.bc.ca<br>juwong@sd43.bc.ca

This course recognizes the need for analytical expertise in a world where innovation is increasingly dependent on a deep understanding of mathematics. This course includes topics that are both traditionally part of a pre-university mathematics course (for example, functions, trigonometry, calculus) as well as topics that are amenable to investigation, conjecture and proof. The course allows the use of technology, as fluency in relevant mathematical software and hand-held technology is important regardless of choice of course. However, Mathematics: analysis and approaches has a strong emphasis on the ability to construct, communicate and justify correct mathematical arguments.

Problem-solving is central to learning mathematics and involves the acquisition of mathematical skills and concepts in a wide range of situations, including non-routine, open-ended and real-world problems. Having followed a DP Mathematics course, students will be expected to demonstrate the following.

1. Knowledge and understanding: recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.
2. Problem-solving: recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems.
3. Communication and interpretation: transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation, use appropriate notation and terminology.
4. Technology: use technology, accurately, appropriately and efficiently both to explore new ideas and to solve problems.
5. Reasoning: construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions.
6. Inquiry approaches: investigate unfamiliar situations, both abstract and real-world, involving organizing and analyzing information, making conjectures, drawing conclusions, and testing their validity.

Group 5 aims: The aims of all mathematics courses in group 5 are to enable students to:

1. Develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power
2. develop an understanding of the concepts, principles and nature of mathematics
3. communicate clearly, concisely and confidently in a variety of contexts
4. develop logical and creative thinking, and patience and persistence in problem solving to instil confidence in using mathematics
5. employ and refine their powers of abstraction and generalization
6. take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities appreciate how developments in technology and mathematics have influenced each other
7. appreciate how developments in technology and mathematics influence each other appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives
8. appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics
9. appreciate the universality of mathematics and its multicultural, international and historical perspectives
10. appreciate the contribution of mathematics to other disciplines, and as a particular "area of knowledge" in the TOK course
11. develop the ability to reflect critically upon their own work and the work of others
12. independently and collaboratively extend their understanding of mathematics

## The Concepts:

The DP mathematics courses identify 12 fundamental concepts as shown below. Explanations of each of these concepts in a mathematical context have also been provided.

| Approximation | This concept refers to a quantity or a representation which is nearly but not exactly correct. |
| :--- | :--- |
| Change | This concept refers to a variation in size, amount or behaviour. |
| Equivalence | This concept refers to the state of being identically equal or interchangeable, applied to <br> statements, quantities or expressions. |
| Generalization | This concept refers to a general statement made on the basis of specific examples. |
| Modelling | This concept refers to the way in which mathematics can be used to represent the real world. |
| Patterns | This concept refers to the underlying order, regularity or predictability of the elements of a <br> mathematical system. |
| Quantity | This concept refers to an amount or number. |
| Relationships | This concept refers to the connection between quantities, properties or concepts; these <br> connections may be expressed as models, rules or statements. Relationships provide <br> opportunities for students to explore patterns in the world around them. |
| Representation | This concept refers to using words, formulae, diagrams, tables, charts, graphs and models to <br> represent mathematical information. |
| Space | This concept refers to the frame of geometrical dimensions describing an entity. |
| Systems | This concept refers to groups of interrelated elements. |
| Validity | This concept refers to using well-founded, logical mathematics to come to a true and accurate <br> conclusion or a reasonable interpretation of results. |

## Group 5 (mathematics) grade descriptors

## Grade 7

Demonstrates a thorough knowledge and comprehensive understanding of the syllabus; successfully constructs and applies mathematical arguments at a sophisticated level in a wide variety of contexts; successfully uses problem-solving techniques in challenging situations; recognizes patterns and structures, makes generalizations and justifies conclusions; understands and explains the significance and validity of results, and draws full and relevant conclusions; communicates mathematics in a clear, effective and concise manner, using correct techniques, notation and terminology; demonstrates the ability to integrate knowledge, understanding and skills from different areas of the course; uses technology correctly in challenging situations-makes efficient use of calculator's functionality when required.

## Grade 6

Demonstrates a broad knowledge and comprehensive understanding of the syllabus; successfully constructs and applies mathematical arguments in a variety of contexts; uses problem-solving techniques in challenging situations; recognizes patterns and structures, and makes some generalizations; understands and explains the significance and validity of results, and draws relevant conclusions; communicates mathematics in a clear and effective manner, using correct techniques, notation and terminology; demonstrates some ability to integrate knowledge, understanding and skills from different areas of the course; uses technology correctly in routine situations-makes efficient use of calculator's functionality when required.

## Grade 5

Demonstrates a broad knowledge and good understanding of the syllabus; applies mathematical arguments in performing routine tasks; successfully uses problem-solving techniques in routine situations; successfully carries out mathematical processes in a variety of contexts, and recognizes patterns and structures; understands the significance of results and draws some conclusions; communicates mathematics effectively, using appropriate techniques, notation and terminology; demonstrates an awareness of the links between different areas of the course; makes use of calculator's functionality when required-may occasionally be inefficient.

## Grade 4

Demonstrates a satisfactory knowledge of the syllabus; applies mathematical arguments in performing some routine tasks; uses problem-solving techniques in routine situations; successfully carries out mathematical processes in straightforward contexts; shows some ability to recognize patterns and structures; has limited understanding of the significance of results and attempts to draw some conclusions; communicates mathematics adequately, using some appropriate techniques, notation and terminology; makes some use of calculator's functionality, but perhaps not always when required-may be inefficient at times.

## Grade 3

Demonstrates partial knowledge of the syllabus and limited understanding of mathematical arguments in performing some routine tasks; attempts to carry out mathematical processes in straightforward contexts; makes an attempt to use problem-solving techniques in routine situations; communicates some mathematics, using some appropriate techniques, notation or terminology; occasionally uses calculator's functionality, but often inefficiently; does not always use it when required and may use an inefficient analytic approach.

## Grade 2

Demonstrates limited knowledge of the syllabus; attempts to carry out mathematical processes at a basic level; communicates some mathematics, but often uses inappropriate techniques, notation or terminology; unable to use calculator correctly when requiredquestions exclusively requiring the use of the GDC are generally not attempted.

## Grade 1

Demonstrates minimal knowledge of the syllabus; demonstrates little or no ability to use mathematical processes, even when attempting routine tasks; communicates only minimal mathematics and consistently uses inappropriate techniques, notation or terminology; is unable to make effective use of technology.

## Marks Calculation

For each exam written in class, grade boundaries are set to determine IB scores (a mark from 1 to 7 ). These IB scores are then converted into percentages as follows:

| IB Score | Percentage |
| :---: | :---: |
| 1 | $0-49$ |
| 2 | $70-75$ |
| 3 | $76-85$ |
| 4 | $86-89$ |
| 5 | $90-95$ |
| 6 | $96-97$ |
| 7 | $98-100$ |

1: Functions and Models
1.1 Four Ways to Represent a Function
1.2 Mathematical Models: A Catalog of Essential Functions
1.3 New Functions from Old Functions
1.5 Exponential Functions
1.6 Inverse Functions and Logarithms

2: Limits and Derivatives:
2.1 The Tangent and Velocity Problems
2.2 The Limit of a Function
2.3 Calculating Limits Using the Limit Laws
2.5 Continuity
2.6 Limits at infinity: Horizontal Asymptotes
2.7 Derivatives and Rates of Change
2.8 The Derivative as a Function

3: Differentiation Rules
3.1 Derivatives of Polynomial Functions and Exponential Functions
3.2 Product and Quotient Rule
3.3 Derivatives of Trigonometric Functions
3.4 The Chain Rule
3.5 Implicit Differentiation
3.6 Derivatives of Logarithmic Functions
3.9 Related Rates

4: Applications of Differentiation:
4.1 Maximum and Minimum Values
4.2 Mean Value Theorem
4.3 How derivatives Affect the Shape of a Graph
4.4 Indeterminant Forms and L'Hospital's Rule
4.5 Summary of Curve Sketching
4.7 Optimization Problems
4.8 Newton's Method
4.9 Antiderivatives

5: Integrals:
5.1 Areas and Distances
5.2 The definite Integral
5.3 The Fundamental Theorem of Calculus
5.4 Indefinite Integral and the Net Change Theorem
5.5 The Substitution Rule

6: Integrals:
6.1 Areas between Curves
6.2 Volumes

7: Techniques of Integration:
7.1 Integration by Parts
7.2 Trigonometric Integrals
7.3 Trigonometric Substitution
7.4 Partial Fractions
7.8 Improper Integrals

9: Differential Equations:
9.1 Modeling with Differential Equations
9.2 Direction Fields and Euler's Method
9.3 Separable Equations
9.5 Linear Equations
11.1: Infinite Sequences and Series
11.9 Representation of Functions as Power Series
11.10 Taylor and Maclaurin Series

Worksheets will be provided for the following topics

- Vector Geometry

1. Addition of Vectors - The Zero Vector - Scalar Multiples
2. Position Vectors and the Ratio Formula
3. Vectors in Cartesian 2-Space
4. The Scalar Product of Two Vectors - Orthogonal Projection
5. The Vector Equation of a Line in Cartesian 2-Space
6. Vectors in 3-Space
7. The Equations of a Line
8. The Angles between Lines and Planes
9. The Intersection of Lines and Planes
10. The Distance between Points, Lines, and Planes
-Complex Numbers
11. Addition, Multiplication, and Division
12. Zeros of a Polynomial with Real Coefficients
13. Geometric Representation of a Complex Number
14. Argument
15. Polar Form
16. De Moivre's Theorem
17. Euler's Formula

## The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

## As IB learners we strive to be:

## INQUIRERS

We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.

## KNOWLEDGEABLE

We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance.

## THINKERS

We use critical and creative thinking skills to analyse and take responsible action on complex problems. We exercise initiative in making reasoned, ethical decisions.

## COMMUNICATORS

We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

## PRINCIPLED

We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.

## OPEN-MINDED

We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view, and we are willing to grow from the experience.

## CARING

We show empathy, compassion and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us.

## RISK-TAKERS

We approach uncertainty with forethought and determination; we work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

## BALANCED

We understand the importance of balancing different aspects of our lives-intellectual, physical, and emotiona-to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live.

## REFLECTIVE

We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.

The IB learner profile represents 10 attributes valued by IB World Schools. We believe these attributes, and others like them, can help individuals and groups become responsible members of local, national and global communities.

